Exercise 8.5.1

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02407 Stochastic Processes

We consider an Ornstein-Uhlenbeck process $\{V_t\}_{t\geq 0}$ with parameters $\sigma^2 = 1$ and $\beta = 0.2$, and initial value $V_0 = 0$. We solve this exercise through a straightforward application of eq. (8.60), i.e.

$$\mathbb{P}\left(V_t \leq 1 | V_0 = 0\right) = \Phi\left(\frac{\sqrt{2\beta}}{\sigma\sqrt{1-e^{-2\beta t}}}\right) = \Phi\left(\frac{\sqrt{0.4}}{\sqrt{1-e^{-0.4t}}}\right) = \Phi\left(\sqrt{\frac{0.4}{1-e^{-0.4t}}}\right),$$

where Φ denotes the distribution function for a random variable following a standard normal distribution. By inserting the different time points, we obtain the values:

$$\mathbb{P}\left(V_1 \leq 1 | V_0 = 0\right) = \Phi\left(\sqrt{\frac{0.4}{1-e^{-0.4}}}\right) = 0.8647,$$

$$\mathbb{P}\left(V_{10} \leq 1 | V_0 = 0\right) = \Phi\left(\sqrt{\frac{0.4}{1-e^{-4}}}\right) = 0.7384,$$

$$\mathbb{P}\left(V_{100} \leq 1 | V_0 = 0\right) = \Phi\left(\sqrt{\frac{0.4}{1-e^{-40}}}\right) = 0.7365.$$

I have used the *pnorm*-function from the R software to calculate the values of $\Phi(x)$.